Models and model uncertainty in the context of risk analysis

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This paper addresses the concept of model uncertainty within the context of risk analysis. Though model uncertainty is a topic widely discussed in the risk analysis literature, no consensus seems to exist on its meaning, how it should be measured, or its impact on the application of analysis results in decision processes. The purpose of this paper is to contribute to clarification. The first parts of the paper look into the contents of the two terms "model" and "uncertainty". On this platform it is discussed how focus on model uncertainty merely leads to muddling up the message of the analysis, if risk is interpreted as a true, inherent property of the system, to be estimated in the risk analysis. An alternative approach is to see the models as means for expressing uncertainty regarding the system performance. In this case, it is argued, the term "model uncertainty" loses its meaning.

Keywords: Risk analysis, models, model uncertainty, risk, uncertainty

1 INTRODUCTION

A common opinion among analysts and decision-makers is that the results of risk analyses are associated with large uncertainties. National legislation, industry standards and company guidelines often require that, if possible, a quantitative evaluation of the uncertainties should be presented as part of the analysis results[1,2]. Model uncertainty is commonly regarded as one of several contributors, and efforts have been made to provide a measure of uncertainty stemming from models applied in the analyses.

Model uncertainty has been subject to considerable attention in recent literature, see for example[3-9]. No agreement seems to exist, however, on the definition of the concept in itself and how it should be measured. Another question, although not so often discussed, is how model uncertainty affects the value of the analysis results when brought forward in a decision-making setting. The intention of this paper is to contribute to clarification of these issues. In order to elaborate on model uncertainty, three basic topics are addressed:

1. The purpose of performing risk analyses
2. What the term "model" means in a risk analysis context
3. The meaning we put into the concepts "risk" and "uncertainty".

Firstly, we state that the main objective of performing risk analyses is to support decision-making processes. Risk analysis enables us to take both certain and uncertain quantities into account and calculate to what extent specific events or scenarios can be expected to occur in the future. Thus risk analysis provides a basis for comparing alternative concepts, actions or system configurations under uncertainty.

Common risk analysis taxonomy contains a large number of expressions like risk modelling, cause and consequence modelling, stochastic modelling, system models and probability models, but what exactly does the concept model mean? In general a model is a simplified representation of a focused aspect of the complex reality. Models applied within risk analysis are mathematical models and enable predictions of future properties of defined systems to be made. Descriptions of uncertainty related to the quantities at a low system level are transformed into uncertainty measures of overall outcomes by means of models representing causal coherence in the system studied.

Different views exist, however, on whether the uncertainties are properties of the world, the models or the analyst applying the model. This disparity of views largely reflects the variety of approaches adopted as analytical basis among members of the risk analyst community, also in terms of fundamentals such as interpretation of risk and uncertainty. As opposed to the classical statistical thinking, which define probability and risk as true properties of nature, the Bayesian approach holds probability as a subjective measure of uncertainty. The choice of basis determines what the probabilities in the analysis input and output express and, as we will argue, also the definition of models and how we should understand and deal with model uncertainty.

The remainder of this paper is organised as follows: In Section 2 principles of modelling in risk analyses and model uncertainty are examined at a general level. Section 3 expands on distinctions between classical and Bayesian approaches to risk and uncertainty. This forms the basis for re-examination of how models and model uncertainty should be interpreted under the different approaches in Section 4. Finally, a summarising discussion and conclusions are included in Section 5.

2 MODELS APPLIED IN RISK ANALYSIS

In risk analyses the extent to which potential undesirable consequences threaten the performance of a given activity is quantified by constructing and analysing a model. The model constitutes a simplified representation of the real system, reflecting the causal relations that produce the events focused on by the decision-makers. The complexity of the model is governed by several factors, such as the complexity of the system, the knowledge about the system that is available to the risk analysis team, the amount of information the decision-makers consider a sufficient basis for making the decision in question and the resources available to the analysis team. The main principle of the analysis is to describe uncertainty related to quantities occurring in the model and derive the probability of the undesirable consequence in question through the model structure by applying the laws of probability calculus.

In this section we will first look into two essential aspects of applying models in risk analysis: representation of causal relations and taking into account uncertainty. Thereafter, some intuitive thoughts about the meaning of and sources of model uncertainty, as well as two approaches to its quantification found in the literature, are discussed.

2.1 Representation of causal relations

Models used in risk analysis usually consist of a system of submodels, describing the system on different levels. Some submodels capture knowledge about how quantities at a low system level influence critical system states. Other take into account how the output risk indices are conditioned on these states. For example, a fault tree model might be applied to describe the possible sets of basic events that produce a defined undesirable event. On a level higher, an event tree structure might represent the conditions under which the undesirable event escalates into a catastrophic event. On the level below the fault tree model, various submodels might describe the factors that determine whether the fault tree basic events occur. A distinction can be made between *quantity-oriented* (physical) and *event-oriented* (logical) models. The remainder of this subsection gives a brief presentation of the main principles of these two categories of causal relation models.

The purpose of quantity-oriented models is to predict the value of an observable quantity Y by expressing knowledge of Y in terms of a set of quantities $\mathbf{X} = (X_1, X_2, X_3, \dots, X_n)$, and the functional relationship f between Y and $\mathbf{X}$, i.e. $Y = f(\mathbf{X})$. The function f is typically established on basis of a mixture of commonly accepted, constitutive models from the fields of physics and chemistry, empirical knowledge, and more intuitive assumptions regarding the systems being analysed. Areas of physics that are often dealt with in risk analyses are:

- Mechanics for calculating accident loads of falling objects, collisions, and explosions, and for determining structural capacity
- Fluid mechanics for evaluating leak-rates of poisonous or combustible fluids and gases and dispersion patterns in air, water, and on the sea surface
- Heat mechanics for assessing the consequences of fires.

An example of a physical model is the expression for speed v of an object dropped from a height h, derived by assuming that the kinetic energy of the object at the reference point equals the potential energy at h:

$$v = \sqrt{2gh}\,, \tag{2.1}$$

where g is the acceleration due to gravity. Another example is the point source model for calculation of radiation from distant fires, see e.g.[10], given by:

$$I = \frac{f\,Q}{4\pi\, r^2}\,, \tag{2.2}$$

where f is the fraction of combustion heat emitted as radiation, Q the total amount of heat released in the flame, and r the distance from the flame.

The equations (2.1) and (2.2) describe the functional relationships between the quantity sought, Y, and some other quantities, $\mathbf{X}$, at a lower system level. The models are applied when $\mathbf{X}$ are easier to predict than Y, and the model assumptions implicitly made comply with the knowledge of the system being studied.

While the quantity-oriented models describe a relationship between a set of factors and the numeric value of a quantity, event-oriented or logical models describe conditions under which *events* occur. Such models are composed of conditions and logical terms, and usually have a binary outcome space, typically 0/1, failure/not failure, etc. Logical models may host physical models or other logical models as submodels. In risk models logical models capture how initiating events at a low system level can develop into scenarios threatening human lives and health, environmental, and economical values. Thus they often deal with the integrity of typical barriers built into the system. Three basic examples of logical models in risk and reliability analysis are:

$$F = g(\mathbf{X}) < 0 = X_1 - X_2 < 0 \tag{2.3}$$

$$F_S = A \bigcup B \tag{2.4}$$

$$F_P = A \bigcap B \tag{2.5}$$

Equation (2.3) describes a failure event F according to a simple load-capacity consideration. F occurs when a load X_2 takes a value higher than the value of the capacity X_1. Equation (2.4) shows the conditions of failure of a two component serial system F_S, i.e. that at least one out of the two components, A or B, fails. Similarly equation (2.5) represents failure of a system of two components in parallel, i.e. failure of both components. An example of a failure event in a more complex system is:

$$F = \left[g_1(\mathbf{X}) < 0 \bigcup\ g_2(\mathbf{X}) < 0\right] \bigcap\ g_3(\mathbf{X}) < 0\,, \tag{2.6}$$

where the g_i functions are based on physical models and system barrier properties. The logical arrangement of the g_i functions by unions and intersections reflects the system barrier philosophy.

2.2 Representation of uncertainty

Application of models like those referred to in Section 2.1 is the analyst's tool for capturing deterministic representations of cause coherence in the system, that is considered essential for the risks being studied. The main concern in a risk analysis, however, is to investigate uncertainty related to possible system states and how this propagates into uncertainty of the defined undesirable events. In this subsection, focus is on how uncertainty is represented when the two categories of models are used.

If a physical model $f(\mathbf{X})$ for prediction of a quantity Y is applied, the uncertainty with respect to Y is expressed through the probability distributions related to the quantities $\mathbf{X}$ in the model. The resulting probability distribution of Y can be determined by an analytical approach, by Monte Carlo simulation or approximate approaches like FORM or SORM described in e.g. Madsen et.al.[11]

In order to simplify assessment of model input and calculations, quantities of which the uncertainty does not impact the results significantly, are in practice often represented solely by their expected values. Another serviceable alternative is sometimes to express uncertainty related to continuous quantities in terms of various kinds of discrete probability distributions.

Logical models describe the relationship between events. Uncertainty is handled in terms of event probabilities. The probability of the event that is modelled (e.g. the top event in a fault tree model) is determined from the input probabilities by accurate mathematical analysis, approximate solutions, or Monte Carlo simulation.

Adding the element of uncertainty to the system model in order to quantify the uncertainty related to the modelled quantity is commonly referred to as stochastic or probabilistic modelling. The terms risk model or reliability models are frequently used about the resulting algorithm.

When a standard probability distribution, such as the normal, Poisson or binomial distribution, is used to express the uncertainty related to a quantity in the model, this is often denoted a probability model.

A topic beyond the scope of this paper is how to quantify uncertainty of the quantities **X**, taking into account mutual dependency between these. A special case subject to discussion in the literature, is the treatment of similar quantities, such as the lifetimes of serial produced units in similar operating environments and systems where repair reintroduces an "as good as new" condition. In a Bayesian framework, see Section 3, such quantities become dependent. The Bayesian solution to the problem is to ensure independence by conditioning on the values of the parameters of the chosen distribution class. We refer to Apeland, Nilsen and Aven[12] and the references therein.

2.3 Model uncertainty

The conception model uncertainty is commonly related to deviations between the real world and its simplified representation in models. When analysing complex systems in real life, compliance between the model assumptions and the properties of the system being analysed never exists in an absolute sense. In most cases the question is rather whether the model can be accepted in spite of infringing one or more of the conditions supporting the model.

As an example, consider application of the model given by equation (2.1) in order to predict the velocity of an object dropped from a crane located on a floating structure. It can then be argued that the vertical motion of the structure due to ocean waves would cause the model to be inaccurate. Such motion would affect the height of fall h, cause an initial speed $v_0 \neq 0$, and a relative vertical motion of the object hit by the dropped object, i.e. three effects that are not taken into account by the model. Another phenomenon not considered by (2.1) that would cause non-compliance for any dropped object is air resistance, which is influenced by the mass and the shape of the object – quantities that are not included in (2.1).

Similar objections can be raised against the radiation model in (2.2). The model assumes that all radiation is emitted from a single point. This is only true for radiation on objects located an infinite distance from the flame. In real cases, where heat transfer is studied within a restricted geographic area, radiation is rather emitted from a surface or a body, i.e. from more than a single geometric point. A second objection can be related to absorption of heat radiation by the atmosphere. The longer the distance to the flame emitting the radiation, the larger is the fraction of emitted energy that will be absorbed. It can thus be asserted that in real cases the model will predict slightly conservative heat transfer effects.

As for physical models, logical models are simplifications and hence inaccurate descriptions of the real world in that they do not necessarily consider all possible causal mechanisms, or take into account all elements of system redundancy.

If representation of uncertainty is regarded as part of the model, inaccuracies may have the form of:

- Wrong probability distribution class
- Inaccurate distribution parameters
- Omission of dependency effects
- Discretisation of continuous quantities or representation in terms of expected value etc.

Intuitively, a distinction can be made between two sources to discrepancies in models:

1. Limitations in the analyst's phenomenon knowledge
2. Deliberate simplifications introduced by the analyst.

Source 1 relates to the abilities of applied science to capture properties of the real world systems and the analyst's knowledge of the governing physical phenomena in the system he looks into. Factors associated with such lack of understanding are typically:

- Highly complex systems and phenomena
- Interaction between human beings and technical equipment
- New systems and phenomena for which few or no models exist
- The quantities considered are associated with the unpredictable conditions governing an unwanted scenario in the future, for example the load-carrying capacities of a marine structure subsequent to a ship impact.

Source 2 contributes to further disparities between the model and the world when the analyst deliberately uses models outside their area of application, i.e. when models are purposely selected to represent real systems with which the model assumptions only partly agree. Motivations for such modelling practice may be:

- Trade-offs forced between project economy and the level of detail in modelling
- The model is considered to serve its purpose sufficiently well in perspective of the overall analysis objectives, in spite of allowing some inaccuracies
- Convenient reduction of the analysis efforts.

Zio and Apostolakis[3] suggest the "adjustment factor approach" to quantify the "error" in model predictions, which can be related to source 1. The principle of this approach is to employ the best model available, denoted Y^*, and compensate for the error associated with Y^* by introducing a factor E. This adjustment factor might be additive (E_a) or multiplicative (E_m), resulting in:

$$Y = Y^* + E_a \tag{4.1}$$

or

$$Y = Y^* E_m \, . \tag{4.2}$$

Another meaning of the term model uncertainty is ascribed to the analyst's doubts regarding which phenomena or physical mechanisms govern the outcomes of interest. Such uncertainty can be handled by the "alternate hypotheses approach" which is also described by Apostolakis[3]. This approach introduces a set of weighting probabilities $\mathbf{P}(Y_i)$, reflecting the relative confidence in a set of models $\mathbf{Y}(Y_1, Y_2, \ldots, Y_n)$, which represent different assumptions about the mechanisms governing the uncertain quantity Y. The models $\mathbf{Y}$ all have a form $Y_i = f_i(\mathbf{X}_i)$, where $\mathbf{X}_i$ is the set of variables in model i, which are described by probability distributions. Each weighting probability $P(Y_i)$ is assigned subjectively by the analysis team and summed up to equal unity, in agreement with the law of total probability.

3 INTERPRETATION AND EXPRESSION OF RISK AND UNCERTAINTY

The risk related to an activity is often formulated as the spectrum of consequences $\mathbf{C}$, discrete outcomes that may follow from so-called undesirable events during the activity, and the associated probabilities $\mathbf{P}$, i.e. (C_1, P_1), (C_2, P_2), … , (C_n, P_n), as illustrated in Fig. 1.

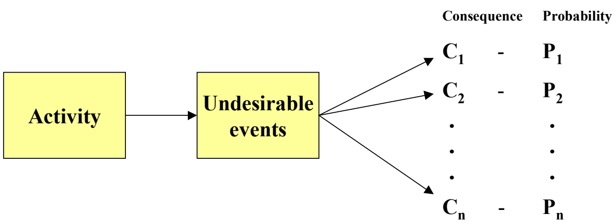

Fig. 1. Risk formulated in terms of defined consequences of undesirable events and related probabilities.

Alternatively, the outcome space of the activity may be given a more general representation through a set of uncertain quantities $\mathbf{Y}(Y_1, Y_2, \ldots Y_n)$. Risk may then be formulated as the simultaneous distribution of $\mathbf{Y}$, $F_{\mathbf{Y}}(\mathbf{y})$.

Given the set of defined consequences $\mathbf{C}$ or outcome quantities $\mathbf{Y}$, our interpretation of the probabilities $\mathbf{P}$ or the distribution $F_{\mathbf{Y}}$, is essential for how we understand the risk picture associated with the activity. Below, we look briefly into three distinct theoretical frameworks.

The traditional approach to risk analysis is based upon principles and methods of *classical statistics*. In this setting, the probability of an event is defined as the relative frequency of event occurrences when the experiment, from which the event develops, is hypothetically repeated an infinite number of times. Hence, taking this approach, the analyst uses risk analysis as a means for estimating true, yet unknown probabilities.

In most branches of the industry subjected to risk analysis, large amounts of experience data are gathered and made available to the analysts. However, the systems being studied are often characterised by tailored system configurations and maintenance philosophies as well as unique operating conditions and working environments. In addition, equipment and operation design is governed by continuous and fast development. The analyst is thus usually left with few or no relevant data for estimation of probabilistic input parameters for the risk analysis models. As a consequence, assumptions that allow the use of the data available are forced, and supplementary information in other forms like expert opinions is widely utilised. Thus, in practical analyses, there is often a conflict between the obvious subjective elements in this estimation practice and the objectivity suggested by the classical statistical approach, by its consideration of probability as an inherent property of the real world. This conflict is often reflected in the difficulties associated with interpretation of probability estimates resulting from analyses of a limited basis of experience data, and with assessment of uncertainty related to such estimates.

The *classical approach with uncertainty analysis*, see e.g. Aven[13,14] and Aven and Rettedal[15], differs from the classical statistical approach primarily by its handling of uncertainty. The uncertainty captured by the classical statistical approach is restricted to the statistical uncertainty in the data applied to estimate probabilistic model input parameters. Considering the strong element of subjectivity in these estimates, it is clear that the uncertainty that arises from statistical variation in historical data only makes a small contribution to the total uncertainty in the estimates. The classical approach with uncertainty analysis allows the uncertainty in the probabilistic input parameters to be expressed in terms of subjective probability distributions, which subsequently may be propagated in uncertainty measures of the final risk indices in the model output. The distributions can be systematically updated when new information arises by the use of Bayes' formula. This approach is also referred to as the *probability of frequency* framework or approach, cf. Apostolakis and Wu[16] and Kaplan[17]. In this framework the concept probability is used for the subjective probability and frequency for the "objective", relative frequency based probability.

We emphasise that the probability of frequency approach is not the same as modern Bayesian statistical theory, as presented by e.g. Bernardo and Smith[18], Singpurwalla and Wilson[19], and Lindley[20]. Sometimes it is however difficult to see the difference in practice. When adopting this theory, focus is on fictitious parameters obtained through thought constructions, and it is allowed for uncertainty distributions of these. The parameters are postulated to be limits of observables, and this is close to the classical thinking.

Common for the classical statistical and the probability of frequency approaches is the focus on estimation of unobservable probabilistic quantities like probabilities and statistically expected values, and on quantifying the uncertainty related to these estimates.

The third framework for interpretation of risk and uncertainty we choose to present here, is a *predictive, Bayesian approach*, which represents a framework for risk analysis putting the attention to observable quantities and using subjective probabilities to express uncertainties related to these quantities. This way of presenting the Bayesian paradigm has been introduced in risk analysis forums in the recent years, see Aven[13], Aven and Rettedal[15]; cf. also Barlow[21], Barlow and Clarotti[22], Bernardo and Smith[18], and Watson[23]. Examples of observable quantities are whether or not an event occurs, the number of fatalities in an accident, and the duration of a delay caused by an undesirable event. Observable quantities express states of the "world", quantities of the physical reality or the nature, that are unknown at the time of the analysis, but will take some value in the future, and possibly become known. In this setting, there exists no true probabilities, and the approach is thus conceptually different from the classical statistical and probability of frequency approaches, in which uncertainty is regarded as a regrettable property of the estimates of presumed true, but unobservable quantities, describing the inherent randomness of the world. The predictive, Bayesian approach will be discussed further in the following section.

4 INTERPRETATION OF MODELS AND MODEL UNCERTAINTY

In this section we discuss the interpretation of models and model uncertainty in light of the approaches to risk and uncertainty presented in Section 3. The question of which parts of the algorithms used to calculate the output risk indices should be regarded as models is first addressed, before discussion of the meaning of model uncertainty and how it may be quantified under the different frameworks.

4.1 The classical statistical approach and the classical approach with uncertainty analysis

Under the classical statistical approach, risk is considered to result from a fundamental or inherent randomness in the natural phenomena of the world (aleatory uncertainty). Probabilistic expressions are used to represent this natural variability. Thus, when models are viewed as simplified representations of reality used to estimate system properties, these expressions must be considered as parts of the models. In other words, when logical and physical models are applied under this approach, they include both deterministic and stochastic elements. The deterministic elements capture known coherence in the system, and the stochastic elements represent uncertainty related to quantities viewed to have a random nature.

The interpretation is quite similar under the classical approach with uncertainty analysis (probability of frequency approach). But, if this framework is applied, the analyst additionally includes representations of uncertainty stemming from his limited knowledge of causal relations, quantities or stochastic elements in the system. However, expressions of epistemic uncertainty should not be seen as part of the models, since it must be ascribed to himself and not to the system represented, see Fig.2.

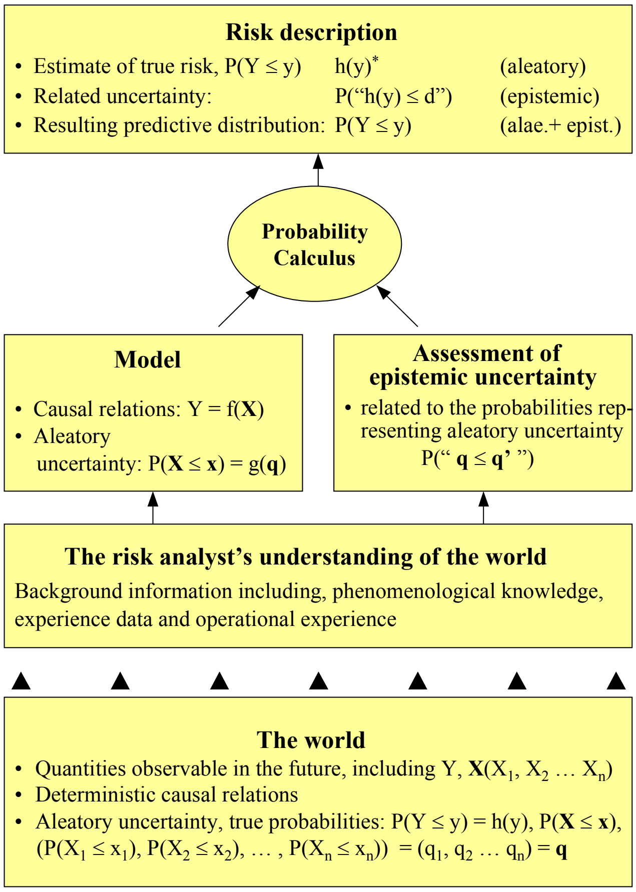

Fig. 2. Interpretation of models in risk analyses under the classical approach with uncertainty analysis (probability of frequency approach). The symbols d and **q'** represent dummy variables. The symbol g represents the function of $\mathbf{q}(q_1, q_2, \ldots, q_n)$, $q_i = P(X_i = x_i)$, that yields the joint distribution of **X**, i.e. $g(\mathbf{q}) = \Pi\, q_i$, if the X_i's are considered independent.

Both aspects of the model, the deterministic representations of cause coherence in the system as well as the probabilistic description of presumed real stochastic elements, deviate to some extent from the real system considered. The deviation may stem from limited understanding of the system (source 1, cf. Section 2.3) as well as deliberate simplifications by the analyst (source 2). Uncertainty about the magnitude of the deviation will be a contributor to uncertainty in the estimate of output probabilities from the model.

The adjustment factor approach to quantification of model uncertainty, described in Section 2, suggests representation of the deviations in an additive or multiplicative quantity E. We do, however, question the reasoning behind this approach. As pointed out by Apostolakis[5] and Buslik[6], if information exists that makes the analyst able to say anything about the error in the predictions by a model, this can be considered simply as a modification of the original model. Furthermore, making efforts to quantify deviations that can be related to source 2, deliberate simplifications, is a contradiction in terms. Simplifications are introduced in order to save resources in some form and are not worthwhile if the time or money saved is spent on analysing the accompanying error. Hence, regarding the technical exercise of quantifying the gap between reality and this modified model, we doubt whether this is at all feasible in practical analyses.

Regarding the alternate hypothesis approach, we see no interpretation of the weighting probabilities, which expresses relative confidence in the candidate models, under the classical statistical framework. The classical approach with uncertainty analysis, on the other hand, allows for the use of subjective probabilities to express degrees of confidence, and would thus comply with this approach. However, we point out that this would imply representation of uncertainty at three levels. First, uncertainty with respect to the nature of the phenomenon considered through weighting probabilities by the alternate hypothesis approach. Secondly, representation of aleatory uncertainty related to the quantities **X** in the model. Finally, epistemic uncertainty in probability distributions related to probability model parameters. Note also that when the probabilities of observable quantities are seen as part of the model, the uncertainty related to the probability model parameter must in itself be interpreted as model uncertainty, see also Buslik[6].

4.2 The predictive, Bayesian approach

From a predictive, Bayesian point of view, all uncertainty is considered a result of lack of knowledge, i.e. it is epistemic. Thus the models, being reflections of the real world, only include the representation of cause coherence relationships between observable quantities, see Fig. 3. The probabilistic expressions are not part of the model, but reflect uncertainty or lack of knowledge of the quantities in the model. Hence, in this framework, terms like stochastic modelling, risk models, uncertainty models, or probability models have no place.

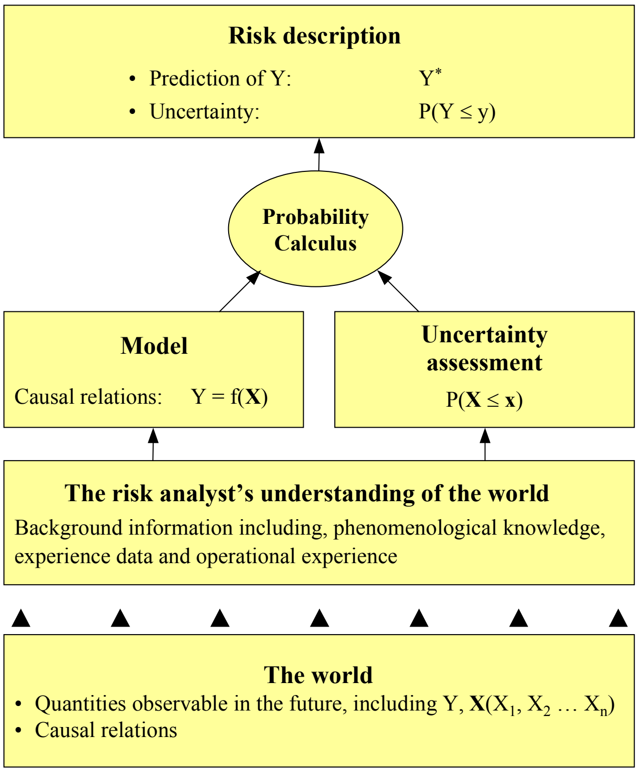

Fig. 3. Interpretation of models in risk analyses under a predictive, Baysian approach.

As emphasised in Section 2, under this approach a model $Y = f(\mathbf{X})$ is a purely deterministic representation of causal mechanisms judged essential by the analyst. It provides a framework for mapping uncertainty about the observable quantity of interest, Y, from expressions of epistemic uncertainty related to the observable quantities **X**, and does not in itself introduce additional uncertainty. In this setting, the model is merely a tool judged useful for expressing partial knowledge of the system. Discussing its correctness, e.g. through the adjustment factor approach, makes no sense.

The alternate hypotheses approach, on the other hand, is a means for weighting confidence in candidate causal mechanisms that might influence the quantity of interest, and is not in conflict with the predictive, Bayesian thinking. However, we consider the notation "model uncertainty" dubious in this context. The resulting mathematical expression does not reflect a lack of precision in the model. It is the ruling cause-consequence coherence that is uncertain, and not the models chosen to represent the candidate hypotheses. The introduction of weighting probabilities simply allows the analyst to differentiate his uncertainty with respect to the nature of the acting phenomena.

Although we find no meaningful interpretation of "model uncertainty" under the predictive, Bayesian approach, in a defined decision-making context there is still room for discussing the goodness or appropriateness of specific models to be used in a risk analysis. If a model is used in given a risk analysis setting, the goodness of the model must have been found acceptable by the risk analyst group. This includes more or less thorough evaluations of the model's accuracy in describing the world, its suitability in the calculations of the risk assessment process, as well as its implications on the decision-making process.

5 DISCUSSION AND CONCLUSIONS

In risk analysis we are concerned with uncertainty related to the outcomes of carrying out some activity that are considered important in a decision-making setting. Examples of outcomes considered are observable quantities such as the number of fatalities, the extent of environmental damage or performance measures of purely economic interest, such as the degree of fulfilment of technical objectives or time consumption. Typical decision processes where risk analyses are used are the overall selection of conceptual lay out in a project, comparison of alternative system configurations, and operational strategies. Risk analyses may improve the decision basis by quantifying the overall risk level associated with the decision alternatives, identifying main contributors to risk, and the most effective measures for reducing it.

Essential in risk analysis methodology is the extensive use of models. Development of models implies trading off complexity to provide a satisfactory representation of the system under study against the simplicity required for analysis purposes. Despite the simplifications and inaccuracies necessarily introduced in this process, the results from analysis of models are often considered to give a valuable contribution to the information basis for decision-making.

As an example, consider a model $f(\mathbf{X})$ used to predict the minimum pressure, P_{min}, occurring during a specific operation in a petroleum well. If the mud circulation pumps stop during this operation, the contribution to well pressure from circulation friction pressure is lost, and the well pressure is temporarily reduced. One cause of losing pump force is power failure, and the number of power failures, X_i, during an operation is one of the quantities included in the model. Failure occurs if P_{min} turns out to be lower than a critical pressure level P_C. A Poisson distribution $Po(\lambda)$, where λ is the mean, is used to represent the uncertainty with respect to X_i. By transforming the Poisson distribution of X_i and the distributions of the other X's to the distribution of P_{min}, the probability of failure is obtained by means of this latter distribution and the value of P_C..

Now, whether the uncertainty with respect to X_i, represented by a Poisson distribution, should be regarded as part of the risk analysis model, depends on the theoretical basis applied. Under a classical statistical or a classical approach with uncertainty analysis (probability of frequency approach), variability is seen as a property of the real system. Being a simplification of the system, the model also includes the probabilistic representation of this variability, in this case, also the distribution of X_i. This thinking is based on the construction of a hypothetical infinite population of well operations similar to the one analysed, and the Poisson distribution is a model of the proportion of outcomes where the number of power failures is less than or equal to a specific value. The Poisson distribution represents aleatory uncertainty, i.e. variation within the population. In our example, the risk analysis is used to estimate $P(failure)$ given by the proportion of the hypotetical population of operations in which P_{min} is lower than P_C. The estimate is associated with uncertainty and assessment of this uncertainty should be included in the analysis. One of the sources to uncertainty is model uncertainty. Hence, according to this point of view, the analyst should provide a measure of the uncertainty that results from the assumptions underpinning the set of models applied in the analysis and the deliberate and unconscious simplifications made. In our example this involves the model f, the Poisson distribution of X_i, and the distributions applied for the other quantities in the model. However, establishing consistent measures of model uncertainty has shown to be difficult. The quantification process is comprehensive and seldom carried out in practical applications.

Alternatively, if the predictive, Bayesian approach is applied as the basis, risk arises from our limited knowledge about the system and is quantified by expressing uncertainty related to observable quantities in terms of probabilities. The uncertainty, e.g. formulated in terms of a Poisson distribution related to X_i, is thus not part of the model, which solely contains the relationship between **X** and the quantity sought, in our case P_{min}. Modelling is a tool allowing us to express the uncertainty in the format found most appropriate to obtain the objective of the analysis. In this case knowledge of power failures is utilised in the assessment of minimum pressure and whether a system failure will occur. The models are seen as part of an apparatus for expressing uncertainty about the system, and do not in themselves introduce additional uncertainty.

Of course, if an infinite or large population of similar operations can be defined, in which X_i belongs, then a distribution function representing a state of nature exists – X_i is an observable quantity, and we can speak about our uncertainty of this distribution function also within a predictive Bayesian approach. In such a case, the Poisson distribution is a model, a representation of the true distribution function. We may refer to this variation in this population as aleatory uncertainty, but still the uncertainty related to the value of X_i is seen as a result of lack of knowledge, i.e. the uncertainty is epistemic. Note that there is a fundamental distinction between uncertainty that involves judgment by the analyst or an assessor, described by subjective probabilities, and uncertainties, or better, variations, that are properties of the world, independent from the analyst.

The question is whether such a population of similar situations should be introduced. In the above Poisson example, it is is not obvious that an infinite or large population of similar operations can be defined. As a general rule, the analyst should, in our view, avoid introducing fictitious populations. This is a tool for expressing the predictive distribution of X_i, cf. Fig. 2, but without a precise understanding of the population, the uncertainty assessments become difficult to perform, and an element of arbitrariness is introduced. For a fictitious population, it is difficult or impossible to assess the goodness of the "model", as relevant data cannot be obtained, without conflicting the requirement of considering similar (relevant) situations. What is a fictitious population and what is a real one is a matter for the analyst to decide, but the essential point we are making here, is that the analyst should think first before introducing a population. There is no mechanical rule saying that aleatory uncertainty exists, and that it should be incorporated in the analysis. The full Bayesian set-up, covering the introduction of a parametric distribution class, specification of a prior distribution for the parameters, Bayesian updating to establish the posterior distribution, and the calculation of the predictive distributions, is a useful tool for coherent analysis of statistical data, but should not be used when not needed.

Let us summarise the main message of this paper. Under a classical statistical or classical approach with uncertainty analysis, attempts to quantify deviations between the real world and its representation in the models might be reflected implicitly in the output probabilities obtained in the analysis or as some uncertainty band related to these. In our opinion such analysis of "model uncertainty" does not add any value to the risk analysis. Rather it diverts the attention away from what is uncertain, namely the outcome of the activity being studied. The contribution of a risk analysis to the decision-making process is to clarify or reduce the uncertainty related to activity outcomes by systemising relevant information. We claim that quantification of model/reality discrepancies also counteracts clear communication of the actual findings of the analysis to the decision-maker.

The interpretation of models and views on "model uncertainty" reflected in this paper, does not imply that the analyst may be indifferent about the degree compliance between the model and the real world. The analysis group and those verifying their work have the responsibility to employ their best engineering judgement to ensure that the differentiation of the models used is adequate for the decision process in question.

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