

Model description and model selection in the context of risk analysis

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Summary

In risk analysis there is an extensive use of models for the assessment of phenomena like ignition, fires and explosion. The choice of models is just as important to the results as the input data used in the models. Due to a constant development towards more sophisticated methods and models it has become difficult for decision makers, and to some extent the risk analysts, to select the most appropriate models to assess the issues they are considering.

When sophisticated models are applied, a large portion of the available resources is often spent on the modelling itself, while little is left for identification and evaluation of risk reducing measures. Furthermore, lack of flexibility in the models sometimes requires dubious assumptions to be made in order to fit the object of analysis to the models and not the other way around.

This paper presents a framework for describing models within the context of risk analysis. By using this framework, resemblances and differences between alternative models representing the same phenomenon may be assessed. On this basis the goodness and the appropriateness of the models may be discussed in a structured way.

With respect to decision making, where the results from the risk analyses is considered to be an important part of the information basis, it is our belief that the framework could be of use to risk analysts as well as decision makers to evaluate and select between candidate models to be used in the risk assessments.

1 Introduction

One of the main reasons for performing a risk analysis is to get better knowledge and understanding of the phenomena or system being analysed in order to make decisions. Many companies that are using risk analysis as a basis for their decision making (hereafter refer to as the clients) do not have the opportunity or capability to perform the risk analysis themselves. They are therefore often using other companies or personnel (hereafter refer to as the risk analyst) to either assist them or to perform the entire analysis for them.

The way a risk analysis is carried out and the quantities that are included and reflected in the analysis is decisive with respect to the decisions that can be made based on the analysis. An important task prior to the execution of an analysis is therefore to identify what the client wants to take decisions about. In other words, we have to identify the purpose of the analysis. However, the way the analysis is carried out is also dependent on the available ways of doing the analysis (including the available models) and the available resources (time and money). The choice of how the analysis is carried out is therefore often a compromise between these and other factors.

There are several choices that have to be made with respect to the way the risk analysis is carried out. This includes the choice of models. When sophisticated models are applied, a large portion of the available resources is often spent on the modelling itself, while little is left for identification and evaluation of risk reducing measures. Furthermore, lack of flexibility in the models sometimes requires dubious assumptions to be made in order to fit the objective of analysis to the models and not the other way around. The risk analyst does often make these decisions alone without any influence and discussion with the client. The result of this can be that the client gets an analysis that is not appropriate or suitable with respect to his needs.

In order to improve the risk analysis we believe that there should be a better dialogue between the client and the risk analyst prior to the execution of the analysis than what is the case for a lot of the risk analyses performed today. In this paper we will focus on improving the communication between the risk analyst and the client with respect to the description and selection of models.

The interpretation and understanding of the term model is dependent on the approach used as the basis for risk and risk analysis. In Section 2 we will briefly present the predictive, epistemic approach, which we use as the basis for our analysis. After a discussion of models used in risk analysis and the differences in models assessing the same phenomena in Section 3, we will present a framework for describing models in Section 4. Our main focus is to create a framework that describe and present the models in such a way that it can be possible for the client and the risk analysts to understand and discuss the available models in order to select the most appropriate model to be used in their analysis. In Section 5 we will discuss the framework and the choice of models.

2 The predictive, epistemic approach to risk and risk analysis

The classical, statistical (frequentist) approach and the so-called combined classical and the Bayesian approach, cf. [2], are the approaches that are usually used in risk analysis. Under both these approaches, focus in the risk analysis is on establishing estimates of presumed true statistical quantities, like probabilities and failure rates. If sufficient experience data are available, the estimation can be based purely on analysis of these under the classical, statistical approach. If data are scarce the combined classical and Bayesian approach allows use of engineering judgements to establish subjective uncertainty measures related to what the true values of the statistical quantities are. Thus, in both frameworks, uncertainty is related to two levels, i.e. the occurrence of future events *and* the true values of the probabilities and failure rates. The combined classical and Bayesian approach is also referred to as the probability of frequency framework, cf. [3] and [4]. In this framework the name probability is used for the subjective probability and frequency for the “objective”, relative frequency based probability.

We find the above two frameworks for dealing with risk not very suitable for supporting decision making in practice, cf. [2]. We are more attracted by the predictive, epistemic approach.

In the predictive, epistemic approach to risk and risk analysis, focus is on predicting observable quantities, like occurrence, or not, of an accidental event, or the number of fatalities or the magnitude of financial losses in a period of time. Observable quantities express states of the “world”, i.e. quantities of the physical reality or the nature; they are unknown at the time of the analysis, but become (or could become) known in the future. Let Y denote an unknown observable quantity and g the relationship between Y and a vector of unknown observable quantities on a more detailed level, $\mathbf{X} = (X_1, X_2, \dots, X_n)$, such that:

$$Y = g(\mathbf{X}).$$

The function g , which we denote a model, is deterministic. Thus if \mathbf{X} were known, Y could be predicted with certainty, given the assumptions underpinning g . However, in most practical cases, such information is not available, and uncertainty related to the predictions has to be taken into account. In the predictive, epistemic approach uncertainty related to the future values of \mathbf{X} is described through an uncertainty distribution $P(\mathbf{X} \leq \mathbf{x})$, $\mathbf{x} = (x_1, x_2, \dots, x_n)$. This uncertainty is epistemic, i.e. a result of lack of knowledge. Then, uncertainty related to Y can be described through a distribution given by:

$$P(Y \leq y) = \int_{\{\mathbf{x}: g(\mathbf{x}) \leq y\}} dP(\mathbf{X} \leq \mathbf{x}).$$

The model g is a simplified representation of the world, and it is a tool used, allowing uncertainty of Y to be expressed through the distribution $P(\mathbf{X} \leq \mathbf{x})$, reflecting more or less of the available knowledge about the relationship between Y and \mathbf{X} , see Fig 1.

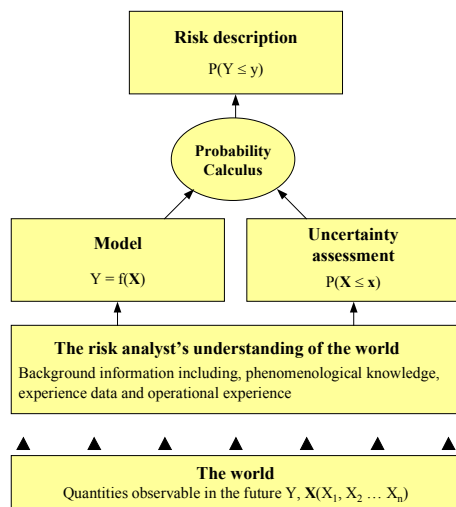


Fig. 1. Basic elements of a risk analysis under a predictive, epistemic approach

Risk related to Y is described through the entire uncertainty distribution $P(Y \leq y)$. Summarizing measures such as the mean and the variance are risk measures that can give more or less information about the risk. The background information used is reported to the decision makers along with the presentation of the risk measures. The models are a part of the background information. All probabilities are conditional on the background information.

An observable quantity represents a state of the world. Thus it includes also quantities that would have been better described as potentially observable. Consider for example the number of injuries. Provided that a precise definition of an injury has been made, there exists a correct value. The fact that there could be measuring problems in this case – some injuries are not reported – does not change this. The point is that the true number exists and according to the definition of an injury and if sufficient resources were made available, that number could be found.

Now, what about a relative frequency probability? Is such a probability observable? Well, the answer is both no and yes. As an example, consider a case where the system is a production facility and we focus on the occurrence of an accidental event (suitably defined) for a one year period. Then we can define a relative frequency probability by the portion of similar production facilities where this event occurs. If this population of similar production facilities is just a thought experiment, it is fictional, then this relative frequency probability is not observable. We will not be able to observe the probability in the future. If, however, such a population can be specified, the probability can be viewed as observable - it is possible to measure it in the future. However, such a population is difficult to imagine in this case unless that we extend the meaning of similar to include “every” type of production facility. Then we would be able to obtain a value of the portion of facilities where this event occurs, but that portion would not be very relevant for the system we study. What is a real population and what is a fictional population needs to be determined in each application. As a general rule we would say that real populations naturally exist when we deal with repeatable games, controlled experiments, mass produced units and large physical populations like human beings, etc.

As a final remark related to a quantity being observable, consider the volume produced in a gas production facility, measured in some unit during a certain period of time, say one year. For all practical purposes, saying that this volume is for example 2.5 would be sufficiently accurate. If you go into the details, the exact production volume could be somewhat difficult to define and measure, but thinking practically, and using the conventions made for this kind of measurements, the correctness of the measurement, for example 2.5, is not an issue. If it were, more precise measurements would have been implemented.

The framework or basis for risk analysis that is referred to as the predictive, epistemic approach is much more than subjective probabilities and Bayesian inference. It relates to fundamental issues like how to express risk and uncertainty, how to understand and use models, and how to understand and use parametric distribution classes and parameters in a risk analysis setting. Compared to the more traditional approaches for risk analysis in the engineering community, the predictive, epistemic approach gives different answers to all these issues.

We define probability as a measure of uncertainty, which means reference to a certain standard such a drawing a ball from an urn. We do not link the definition to gambling situations with prizes and decision making, as is often done in the literature: When a person says that a probability of occurrence of an event A , is $P(A)$, he implies that he is willing to pay (assuming linear utility for money) $P(A)$ now in exchange for 1\$ later if event A occurs. To us such a definition of probability is not so attractive, it complicates the assignments as it introduces more dimensions; decision making involving money.

For a further description of the predictive, epistemic approach, see [1].

The remaining part of this paper is based on the predictive, epistemic approach to risk and risk analysis. Some comments and remarks will however be given with respect to other approaches as well.

3 Models used in risk analysis

In practice, when we are using a model in risk analysis today we are using a computer version of the model. A computer version of a model can for example be a stand-alone software program or a programmed spreadsheet. As we will discuss later in this paper, one of the decisive factors when selecting a model is in many situations the time and cost related to using it. This is however not easy to identify by only evaluating the model itself. Hence, in a practical context we have to evaluate the computer versions of the models in order to be able to select the most appropriate models for our assessment. The choice of models is

therefore often conditioned on the available computer versions and not only on the models themselves. Hence, if not stated otherwise we mean the computer version of a model when we are talking about a model in the remaining part of this paper.

3.1 Differences in models assessing the same phenomena

There can be several models available for the assessment of one specific phenomenon. With respect to the models themselves, and not the computer versions of the models, the differences can be divided into three types or categories.

3.1.1 Different understanding or interpretation of the phenomena

As for many phenomena in the world there can also be different theories, understanding and/or interpretation of a phenomena included in risk analysis. The understanding and/or the interpretation of the phenomena are decisive for which quantities that are included in a model and for the functional relationship between the quantities.

3.1.2 The level of details

Models may also be different with respect to the level of details included in them. While the prediction and the uncertainty related to an observable quantity X_i may be input in one model, a sub-model $X_i=f(W_1, W_2, W_3)$ could be used to represent the quantity X_i in another model. In the second model the input would be predictions and uncertainties related to the quantities W_1, W_2, W_3 and not the quantity X_i . Whether a model is a main-model or a sub-model depends on what phenomena we are assessing. An ignition model can for example be a main-model if we explicitly are assessing ignition, or it can be a sub-model if it is included in a model that assess the number of fatalities due to fires from a process leak. The definition of a model given in Section 2, is however applicable for both main-models and sub-models.

3.1.3 The use of assumptions or predictions and uncertainties

Some observable quantities may be represented by fixed values in some models and by predictions and probabilities representing the uncertainty about the quantities in other models. For example: The number of leaks in a period of time may be given as a fixed value 5 in one model and as range of possible outcomes 0, 1, 2, ..., 20 with associated probabilities in another model. We refer to the use of fixed values as assumptions in the following discussion.

4 A new framework for description of models

The description of models that are used in risk analysis can vary a lot with respect to what is included in the description and how it is presented. It is therefore not easy to generalize with respect to how such a description is usually given. However, such a description is often a "user manual" that focuses on how the model should be used. What is often missing is a description of when and for what situations/scenarios the model is considered usable. This is also related to a description of the limitations in the models that also often is missing.

We believe that the risk analysis can be improved if the client is more involved in the process of selecting the way the analysis is carried out. By discussing the alternatives prior to the execution of the analysis, the client will get a better understanding of what his options are and to understand what he gets by choosing one alternative instead of another. In order to make the client able to participate in the selections, the possible ways of doing the analysis has to be described in a way that makes it understandable for both the client and the risk analyst. As a contribution to this we have established a framework for describing models used in risk analysis.

The purpose of the framework is to structure the presentation and description of models in a way that makes it possible for the client and the risk analyst to discuss and select the most appropriate model for their assessment. By using the framework

it will be easier to understand the strength and weakness to each model and to understand their differences. If a client is considering several risk analysts, than the description will also make it easier to understand the differences (with respect to the models used in the analysis) of choosing one risk analyst instead of another. Hence, the way the analysis is performed will be more prominent to the choice.

The basis for the framework is the predictive, epistemic approach for risk and risk analysis, ref. Section 2. However, we do not see any problems by using the framework to describe models that are based on other approaches like the classical, statistical (frequentist) approach and the combined classical and Bayesian approach. A description of models by the use of this framework will therefore improve and simplify the understanding and selection of models regardless of the approach the model is based on. The understanding of what a model is and the quantities included in a model will of course be dependent of the approach. Terms like uncertainty and model uncertainty will therefore have different meanings. This will of course affect the description of the models.

The framework has been created with respect to models used to assess phenomena like leaks, ignition, fire, explosions etc. However, we see no problems by using the framework to describe other models as well. As we mentioned in Section 3, the choice of models is today in most situations choice of computer version of models. The framework is therefore created to describe computer versions of models.

The proposed framework contains the following parts:

1. Description of the phenomena
2. Brief presentation of the available models
3. Detailed description of each model. This includes a description of the:
 - Model structure
 - Area of application
 - Limitations
 - Required resources
 - Input data and output data
 - Communication with other models
 - User interface
 - Technical documentation

The framework is presented below. It describes how a risk analyst (company) should present its models for the assessment of a specific phenomenon. We have included some remarks with respect to how the framework differs from how a description of models often is given in practice today, cf. e.g. [5], *Here ref.*

4.1 Description of the phenomena

Before each model is presented there should be a common description of the phenomena. This should be a non-detailed and non-technical description that makes it understandable for persons without detailed knowledge about the phenomena. The description should include a presentation of the majority of know quantities that affect the phenomena. A description of the relationship between the quantities should however not be given here. This should be included in the description of each model.

Remarks

Based on our experience such a common description of the phenomena is usually not given. The presentation of the phenomena is usually limited to each specific model. Potential users of the models that do not have detailed knowledge about a phenomenon may therefore get the impression that the model actually represents the full complexity of the phenomena. By having a common description of the phenomena before each model is described the user will get a better understanding of the complexity in the phenomena.

4.2 Brief presentation of the available models

It should be possible for the potential users to identify whether a model is suitable or not for their assessment without having to

review it in detail. A brief presentation of the available models should therefore be given. This should be a summary of the detailed description, and it should include (for each model): what the model does, area of application, required resources (time and cost etc.), limitations, what input data it needs and a description of the results. Based on this presentation only, the users should be able to sort out those models found applicable for further evaluation. A model for the assessment of internal ignition is here given as an example of a brief presentation. The description of the input data, output data, and required resources is excluded in the example.

4.2.1 Example: Time Independent Internal Ignition Model

The Time Independent Internal Ignition Model (TIIM) assess internal ignition within a module without taking time and the size of the module into consideration. Based on the lower flammability limit (LFL), the ventilation within the module and the presence of ignition sources in the module the model predicts whether a leak will ignite or not. The model has been programmed into Microsoft Excel. The number of time ignition will occur or not for 100, 500 and 1000 simulations are calculated by the use of Monte Carlo simulations. A probability describing the uncertainty related to the occurrence of ignition or not is calculated for all simulations. The quantities used as input data in the model are given by predictions, and probabilities representing the uncertainty regarding the quantities.

Area of application

The model is considered to be applicable for coarse evaluations at the early design phase of an installation/plant. The model enables the user to include three different probability distributions for the quantities: leak rate, ventilation rate and lower flammability level. This makes the model flexible with respect to the evaluation of these quantities' effect on the results. It also enables the user to assess fixed values for some of the quantities without changing the other input data (for example: fixed leak rate of 5 kg/s).

Limitation

The model does not take the time aspect into consideration. Hence, it is not possible to reflect any variation in the leak rate (due to isolation and/or blow down and the volume of the leaking segment), ventilation rate and shut down of ignition sources etc. as a function of time. This could however be partly solved by running the model several times with changed input data each time. The model uses a coarse approach to calculate the uncertainty related to whether the concentration in the module is flammable or not. Only the release rate, ventilation rate and the lower flammability level are included in the calculations. Hence, neither the size of the module, the amount released, blow-down and isolation of the leaking segment etc. are included. The model does not reflect the location of the equipment in the module. Hence, neither the location of the leak or the placement of the ignition sources can be reflected.

Remarks

A summary and brief presentation of the available models is usually not given in the presentation of models. The description of each model is often given in a way that makes it difficult to compare them. Nor is it usual to focus on the area of application and the limitation in such a description.

4.3 Detailed description of each model

After the brief presentation of all the available models, each model should be described in more detail. The detailed description should include the following.

4.3.1 Description of the model structure

The first part of the detailed description should describe the model itself. The understanding and interpretation of the phenomena, the level of details and the use of assumptions or predictions and uncertainties, ref. Section 3.1, should be presented. All quantities and the deterministic functions included should be described. This includes a description of: the main-model, sub-model(s), the input data and the results (output). A

graphical presentation of the model structure could be used in addition to the verbal description. Such a presentation can make it easier to identify and understand the model structure (main-model and sub-model(s)) and the relationship between the quantities included. If a graphical presentation is used, it should also be possible to identify whether the quantities given as input data are assumptions (fixed values) or not.

4.3.2 Area of application

The main reason for using a model should be that it is found applicable and appropriate for the assessment of the current situation or scenario. In order to identify if a model is appropriate or not there should be a description of the situations and scenarios that the model is considered to be suitable for.

Remarks

Based on our experience, such a description is sometimes given in the description of models. The description is however often given by the creators of the models. This may sometimes result in an exaggerated positive description that can give the impression that a model is suitable for all situations and scenarios. The risk analyst should describe the models themselves based on their own experience. We do not believe that the main objective for the majority of risk analyst is to persuade the clients to use a specific model (for example the most "advanced" or complex model) in all situations. This framework does however not prevent them for doing so. The objective is to use the most appropriate approach (including the most appropriate models) given the prevailing conditions. This framework will improve this, due to that it forces the risk analyst to describe when and for what situations each model is suitable for and to explain why it is so.

4.3.3 Limitations

All known limitations to the model should be described. By limitations we here mean what cannot be reflected in the model. The implication of the limitation with respect to how it affects the results etc. should also be described. For example: Some models that assess ignition or fires assume a stoichiometric mixture of gas (methane) and air, cf. [5]. The implication of this, compared to the mixtures that actually can occur in the event of a leak on a specific installation, is however rarely described.

Remarks

A presentation of the limitations is usually not given as a part of the description of models. To know the limitations is in our opinion just as important as knowing the area of application of a model. The description of the limitations should supplement the information given in the area of application.

4.3.4 Required resources

The description should present the time and cost etc. required to use the model. This should be presented in a way that makes it possible for the users to identify the total use of resources. For example: If the model uses simulations, it should describe the time and cost per simulation and the number of simulation required. Additional costs by using the model should also be described. The use of resources will depend on several quantities (the number of scenarios modelled, the number of simulations per scenario etc.). However, the description should present how the use of resources could be predicted if these quantities are identified. The description should also include requirements (technical skills etc.) to the user.

Remarks

A presentation of the required resources is sometimes given as a part of the description of models. This information is however often incomplete with respect to the total use of resources related to using the model.

4.3.5 Input data and output data

The description should present all input data required and all output data (results). This should include a description of what

the quantities are and their type and form. Graphs and tables should also be described.

4.3.6 Communication with other models

The output data from a model may be designed in a way that makes it easy to use the data as input data in other models. This makes the communication between models easier. If the model has this property (with respect to the input data or the output data) than this should be described.

4.3.7 User interface.

Although it may not be found important to the decision maker, the user interface to the model should also be described. This is to ensure that the user understand how to use the model and to ensure that the input data is given in the correct location etc.

4.3.8 Technical documentation

The technical documentation should include what the software/calculation tool does and how it performs its calculations. If for example the model is a programmed spreadsheet in Microsoft Excel, than it should not be given a description of Microsoft Excel but only a description of the programmed model. However, references should be given to a further description of the tool (e.g. references to Microsoft Excel). The description should also include the technical requirements for the model (i.e. computer requirements, software requirements, etc).

5 Discussion

5.1 Reasons for using the framework for describing models

Companies that are operating in the Norwegian part of the North Sea today are using a large amount of resources on risk analysis. To successfully implement and use a common framework for describing models these companies have to understand and agree upon the need for such a framework in order to improve their risk analysis. These and other companies that are using risk analysis could make a demand to the risk analyst (companies) to describe their models based on this framework. This could for example be a “standard” for describing models. If all the companies that are performing risk analysis were using the same framework it would give several advantages for the clients. It would first of all make it easier to identify the most appropriate models to be used in their analysis. The results of this could also be that the client identify that there are no appropriate models available. For example: A client wants to identify where he should locate some new equipment in a module based on the risk related to fires and explosion. If there are no models available that can reflect the location of the equipment, than he should not use any of the models. The alternatives are then to develop a new model or to use another approach for the assessment. Secondly, it would be easier for the client to identify the differences between the risk analysts (companies) that they are considering to use when performing the risk analysis. Instead of focusing on the cost of choosing one risk analyst (company) instead of another, the focus will be on which company that have the most appropriate models available for the assessment. Third, after reviewing models for one assessment the client would be familiar with the way the models are described if he has to perform other risk analyses later on. Forth, the risk analyst will be forced to understand and know the advantages of using one model instead of another. Arguments like “you should use this model because it is the newest and most advanced model available” should not be given any weight if the risk analyst can not explain why and in what way the model is better than other models. Hence, instead of always using the latest and most advanced models, it will improve the selection process to focus on the clients needs.

5.2 Selection of models

When a client and a risk analyst are deciding on how they want to perform a risk analysis there are several conditions that will influence on the choices. The selection of models can therefore not be regarded as an isolated task independent of these

conditions. In the following we use the term “current settings” for all the demands and constrains that are prevailing for each specific risk analysis. In order to identify the current settings and to identify the most appropriate approach for the analysis the following issues should be examined:

What is the purpose of the analysis?

This may be seen as an unnecessary question with an obvious answer. It is however our opinion that this is not the case in many situations. The client may have an initial idea or opinion of what he wants. However, without having the overview on the available options he may not know what his options are. When analysing a specific issue or phenomena, e.g. ignition, there may be several known and unknown quantities that may be of interest to the client. Some quantities may be relevant at an early stage, while others may be relevant only at a later stage. An example of this could be risk-reducing measures that may be relevant to identify at a later stage if the risk is considered to be unacceptable. In order to identify the purpose of the analysis the risk analyst and the client should discuss this issue thoroughly with respect to the current situation and also any further need the client may have. This may increase the possibility of choosing the most appropriate approach and models.

What background information is available?

All models need some sort of input data. The required amount and level of details can however vary a lot. An important task is therefore to evaluate whether all the required input data for a specific model are available in the current settings.

What have to be reflected or included in the model?

All models are a simplification of the real world and will never be able to reflect all quantities effecting a specific issue or phenomena. The number of quantities reflected or included in a model can however vary a lot. An important task is therefore to evaluate which quantities the client wish or requires being included in the model. For example: Does the model assess the local condition in an acceptable way, or is the model too general? When the number of quantities reflected in a model increase, the amount of resources used on the modelling itself will in most cases increase as well. Hence, the choice of models is often a compromise between theses and other prevailing conditions.

What are the available resources?

Available resources will in several situations be of decisive importance. It is our opinion that this should not be the first issue to address when selecting a model. By reviewing the questions above before considering the available resources the client may identify several cost and time saving effects when reviewing e.g. the overall lifetime of the installation/plant. This may change his use of resources at the current time. Hence, this may change what is considered to be the best model to be used. The available resources will also be decisive with respect to how much time and money that can be used on each task included in the whole assessment. The use of the available resources should be evaluated for the entire analysis or assessment and not separately for each issue or phenomena included in the assessment.

After identifying the current settings the client and the risk analyst will have a good basis to identify and select the most appropriate approach for performing the risk analysis. If the available models are described in the way we have proposed in the framework presented in this paper, we believe that it will help the client and the risk analyst in their selection of the most appropriate models.

5.3 Evaluation and recommendations

The framework presented in Section 4 should be tested and developed further. It should be tested for models describing different phenomena in order to make it usable for all models used in risk analysis. To ensure that both clients and risk analysts consider the framework to be a good foundation for selecting models, representatives for them both should be involved in the further development. It is our intention to carry out a research

project where we will develop the framework further. This work will include a full description of several models for several phenomena.

6 Conclusion

We believe that the framework for describing models presented in this paper will improve the risk analysis. By using this framework, resemblances and differences between alternative models representing the same phenomenon may be assessed. On this basis the goodness and the appropriateness of the models may be discussed in a structured way with respect to the specific situation and/or system being analysed.

With respect to decision making, where the results from the risk analysis is considered to be an important part of the information basis, it is our belief that the framework could be of use to the risk analyst as well as the decision makers to evaluate and select between candidate models to be used in the risk assessment.

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